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THESIS

COST ESTIMATING RELATIONSHIPS FOR
FIGHTER AIRCRAFT

by

Hong, Won Pyo

March 1987

Thesis Advisor   Michael G. Sovereign

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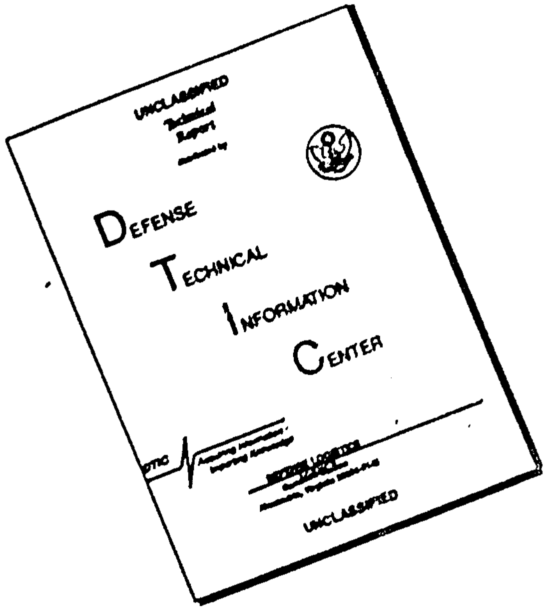

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution is unlimited
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL (If applicable) 55	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000	7b ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO / PROJECT NO / TASK NO / WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

COST ESTIMATING RELATIONSHIP FOR FIGHTER AIRCRAFT

12 PERSONAL AUTHOR(S)

HONG, Won Pyo

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Master's Thesis	FROM ____ TO ____	1987 March	60

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Cost Estimating Relationships, Fighter Aircraft, Price-Level, Cost-Quantity, Ordinary Least Squares (OLS) Estimation

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This thesis presents Cost Estimating Relationships (CERs) for fighter aircraft. Since the fighter aircraft is one of the most important tactical weapon systems, it is very useful to establish CERs solely for fighter aircraft. Using the public data on U.S. fighter aircraft, Ordinary Least Squares(OLS) is used as the primary statistical method of establishing CERs. The data collection techniques and adjustments used are discussed, and simple and multiple linear regressions are performed on various combinations of the explanatory variable. This thesis then shows that CERs based on new fighter aircraft data are more reliable than those based on new and old fighter aircraft data.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL Michael G. Sovereign	22b TELEPHONE (Include Area Code) 408-646-2618 / 22c OFFICE SYMBOL 55Sm

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Approved for public release; distribution is unlimited.

Cost Estimating Relationships for Fighter Aircraft

by

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Major, Korean Air Force
B.S., Korean Air Force Academy, 1977

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1987

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ABSTRACT

This thesis presents Cost Estimating Relationships (CERs) for fighter aircraft. Since the fighter aircraft is one of the most important tactical weapon systems, it is very useful to establish CERs solely for fighter aircraft. Using the public data on U.S. fighter aircraft, Ordinary Least Squares (OLS) is used as the primary statistical method of establishing CERs. The data collection techniques and adjustments used are discussed, and simple and multiple linear regressions are performed on various combinations of the explanatory variables. This thesis then shows that CERs based on new fighter aircraft data are more reliable than those based on new and old fighter aircraft data.

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ACKNOWLEDGEMENTS

I would like to express my gratitude to my country, the Republic of Korea, and particularly the Korean Air Force for allowing me the opportunity to study at the Naval Postgraduate School.

Additionally, I want to give my sincere thanks to Professor Michael Sovereign for his advice and willing support in the completion of this work. I also owe my thanks to Professor Dan Boger for his significant and useful suggestions.

Finally, I want to give a special thanks to my wife, Ok Woon, for her devoted assistance during our stay in the United States.

I. INTRODUCTION

A parametric cost estimate has been defined as an estimate which predicts cost by means of explanatory variables such as performance characteristics, physical characteristics, and characteristics relevant to the development process, as derived from experience on logically related systems [Ref. 1: p.72]. It is based on the assumption that the past is somehow a reliable guide to the future, which means the estimation captures the relationship between past experience and future application.

The cost estimation of military hardware uses experience on existing equipment to predict the cost of next-generation weapons. Traditionally, acquisition of next-generation weapons requires substantial costs. In the past, however, cost was not always a major consideration in choosing the weapons. To save money in the long-run and operate within a tighter budget, costs must be reliably estimated during requirements formulation in determining which weapon provides the best value in fulfilling mission needs.

Cost Estimating Relationships (CERs) are mathematical equations which relate system costs as a function of various explanatory variables. They are most generally derived through statistical regression analysis of historical cost data. The construction and use of CERs forms the foundation for making independent parametric cost estimates [Ref. 2: p.2].

A. THESIS OBJECTIVE

Developing new CERs for fighter aircraft is the major objective of this thesis. In fact, there are several cost estimating methods and CERs for aircraft. This thesis will discuss the statistical approachs and the CERs for fighter aircraft only using explanatory variables such as thrust, weight, etc.

This thesis also has objectives related to the goal of developing new CERs. They are:

1) To research currently developed CERs based on historical data. There are many CERs which were developed in previous periods. They may be used by an experienced analyst and study of them will be helpful to develop new CERs.

2) To present data collection and adjustment approaches. Collecting the right data and adjusting the collected data are required in order to develop CERs. Data imperfections are frequently encountered difficulties in weapon system cost estimation.
3) To apply alternative statistical methods. CERs that use explanatory variables are relied upon to predict the cost at a high level of aggregation. The statistical techniques can be used in a variety of situations, but not for all situations. They will vary according to the purpose of the study and the information available.
4) To apply CERs. By using newly developed CERs, it may be possible to predict the costs of fighter aircraft. Also, it may be possible to estimate the costs of international fighter aircraft from this CER.

B. WHY DO THIS ?

Korea (South) knows the misery of war as a result of the Korean War (1950-1953) and wishes to live in peace forever. However, North Korea is a belligerent communist country. Therefore, as a deterrent to an all-out war, Korea has to have high defense capabilities. Maintenance of a strong defense force is one of the most reliable ways to keep the peace.

Ownership of superior weapon systems is one of the best methods of maintaining strong defenses. Fighter aircraft are one of the most powerful weapon systems developed for modern warfare. However, fighter aircraft acquisition is extremely expensive. Since excessive spending for defense will check national development, the choice between systems must be seriously considered.

Korea is still a developing country and is currently one of the major weapon importing countries. Nevertheless, the economic growth of Korea is worthy of close attention. Korea's economy has been growing at an increasing rate for more than twenty years. As a result, Korea is now changing from a weapon importing country to a weapon producing country.

At this time, it could be meaningful to develop new CERs for fighter aircraft. CERs are based on readily available explanatory variables, so they allow the decision maker to evaluate the cost impact of future designs and make trade-offs accordingly. After acquisition, the potential use of these CERs still exists. They may be used as validated CERs the next time. However, since the earlier CERs are out of date in that they did not include the newest data, developing new CERs is necessary.

Korea's particular interests regarding fighter aircraft are weight, speed, and electronic equipment. As a defense force, fighter aircraft must be sufficiently lightweight that they can be used quickly to react against attacking aircraft. However, as interceptors, fighter aircraft have to have high speed capability and superior electronic equipment in order to intercept targets. Therefore, fighter aircraft must be lightweight, yet be able to reach speeds of at least mach 2.0, and carry the newest superior electronic equipment. Fighter aircraft such as the F-16 or F-18, for example, are the most suitable types for Korea.

C. ORGANIZATION

Chapter II introduces some of the CERs that have been developed for aircraft. Chapter III deals with the data collection and adjustment. Chapter IV concerns the statistical approach and includes a discussion of the ordinary least-squares method as a regression technique. Chapter V deals with the analysis of the established models and includes a description of the prediction analysis which estimates the costs of an international fighter aircraft from the CERs of U.S. fighter aircraft. Finally, Chapter VI offers conclusions regarding the interpretation of selected CERs.

II. PRIOR FIGHTER AIRCRAFT CERs

As implied earlier, CERs are based on historical data. These CERs are no better than the data on which the CERs are based. Therefore, reviewing some of developed methods and models may have a beneficial effect on developing new CERs.

Many organizations have developed cost models, and different techniques have been employed. Through the years the Rand Corporation has organized and updated the Department of Defense (DOD) data base for airframe costs, identifying the deficiencies and correcting them where possible, mainly in support of Air Force sponsored research efforts.

"A Computer Model for Estimating Development and Procurement Costs of Aircraft (DAPCA-III)", which was published in 1976, is one of Rand's aircraft airframe cost models [Ref. 3]. It is based on a sample of twenty-five U.S. military aircraft including fighter, attack, bomber, and cargo aircraft. The model uses CERs to estimate the development and procurement costs of two major flyaway subsystems of the aircraft: airframe and engines. Avionics costs are included in the model but are not derived parametrically. These costs, however, do not quite constitute the total system cost of the aircraft.

Table 1 shows the CERs used in DAPCA-III. They are based on the cost of total production quantity of 200 units including prototype aircraft. For those aircraft whose total production quantity is less than 200 units, the cost-quantity relationship or learning curve is used in order to obtain a value at that quantity. CERs used in the model are based on log-linear regressions (they are shown in the power form). The major explanatory variables are airframe unit weight and maximum speed at the best altitude. Additionally, the time of first flight in calendar quarters after 1942 is found to be a significant explanatory variable for recurring manufacturing labor and materials, and improves the statistical properties of the equation. Thus, equations with and without the time variable were considered separately. Also, the dummy variable designates whether cargo or noncargo aircraft were used for flight test cost.

Costs are provided in seven categories: total engineering hours, total tooling hours, nonrecurring manufacturing labor hours, recurring manufacturing labor hours, nonrecurring manufacturing material costs, recurring manufacturing material costs, and flight test costs. All costs used in the model are in constant 1975 dollars.

TABLE 1

SELECTED CERS FROM THE DAPCA-III MODEL

$$E = 20.032 \cdot W^{0.6636} \cdot S^{0.9871} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-6}$$
$$T = 522.39 \cdot W^{0.6214} \cdot S^{0.5323} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-6}$$
$$ML_{NR} = 0.62597 \cdot W^{0.6883} \cdot S^{1.2109} \cdot 10^{-6}$$
$$ML_R = 1188.5 \cdot W^{0.8306} \cdot S^{0.5464} \cdot T^{-0.4711} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-6}$$
$$ML_R = 581.55 \cdot W^{0.7830} \cdot S^{0.4297} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-6}$$
$$MM_{NR} = 0.030614 \cdot W^{0.7240} \cdot S^{1.9240} \cdot 10^{-6}$$
$$MM_R = 93.409 \cdot W^{0.8121} \cdot S^{0.6951} \cdot T^{0.4744} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-6}$$
$$MM_R = 191.85 \cdot W^{1.8600} \cdot S^{0.8126} \cdot 200^{-(b+1)} \cdot Q^{b+1} \cdot 10^{-5}$$
$$FT = 153.25 \cdot W^{0.7095} \cdot S^{0.5856} \cdot Q_{FT}^{0.7160} \cdot DV^{-1.5570} \cdot 10^{-6}$$

where:

E = total engineering hours (millions)

T = total tooling hours (millions)

ML_{NR} = nonrecurring manufacturing labor hours (millions)

ML_R = recurring manufacturing labor hours (millions)

MM_{NR} = nonrecurring manufacturing materials cost (millions of 1975 dollars)

MM_R = recurring manufacturing materials cost (millions of 1975 dollars)

FT = flight test cost (millions of 1975 dollars)

W = airframe unit weight (lb)

S = maximum speed at best altitude (kts)

Q = airframe quantity

b = exponent corresponding to cumulative average learning curve slope

T = time of first flight (calendar quarters after 1942 = $4 \cdot$ [input date $-$ 1942.75])

Q_{FT} = number of flight test aircraft

DV = dummy variable (1 for noncargo, 2 for cargo aircraft)

DAPCA-III is a meaningful model for use as a long-range planning tool for normal, full scale production programs. However, the model is based on a sample of several different types of military aircraft. A cost model based on a more homogeneous data sample is the result of the work of J. Large. It presents a parametric cost model for fighter aircraft only [Ref. 4].

Large's "A Comparison of Cost Models for Fighter Aircraft", which was published in 1977, is another of Rand's aircraft cost models and is referred to as the Large model [Ref. 4]. It derives CERs to estimate the fighter aircraft cost only. There are two types of CERs in the model. One is derived from a sample of seventeen U.S. military fighter aircraft only, while the other is derived from a sample of thirty-one different types of aircraft. The larger sample fighter aircraft data includes several older fighter aircraft as well as new fighter aircraft.

Table 2 shows the CERs based on a sample of fighter aircraft only. They are based on cumulative total production quantity of 100 units. Like DAPCA-III, the most reliable explanatory variables are airframe unit weight and maximum speed. Additionally, the model afforded an opportunity to examine an explanatory variable that was thought to have special applicability to fighter aircraft. It is referred to as the specific power (P) and represented as

$$P = 0.003069 \times \frac{(\text{static thrust})(\text{max speed})}{\text{combat weight}}$$

Both speed and specific power were considered separately along with weight and other variables in the regression analyses, for comparison purposes.

Costs are provided in seven different categories: cumulative total engineering hours, cumulative total tooling hours, development support cost, flight test cost, cumulative recurring manufacturing hours, cumulative recurring manufacturing materials cost, and cumulative recurring quality control hours. Then, in order to accommodate the less detailed older data, two of the cost categories in DAPCA-III -- nonrecurring labor and materials-- are combined into a single category, development support. All costs used in the model are in constant 1973 dollars.

The Large model, as a model based on fighter aircraft only, compares the CERs for fighter aircraft with the CERs for different types of aircraft, and with the CERs used in DAPCA-III. However, since the model was published in earlier times, the cost information for older aircraft are less reliable than for later aircraft, and the development and production experience of these earlier aircraft are not considered an appropriate indicator of the future. Furthermore, as in DAPCA-III, CERs used in the model make use of subsystem characteristics in order to estimate the costs of airframe, engines, etc. Therefore, it would be desirable to develop new CERs which are based on recent aircraft data and make use of overall aircraft characteristics.

TABLE 2

SELECTED CERS FROM THE LARGE MODEL

$E_{100} = 0.000015 \cdot W^{1.14} \cdot S^{1.29}$

$E_{100} = 0.0276 \cdot W^{1.24} \cdot P^{0.72}$

$T_{100} = 0.0583 \cdot W^{0.657} \cdot S^{0.760}$

$T_{100} = 4.754 \cdot W^{0.715} \cdot P^{0.446}$

$ML_{100} = 0.097 \cdot W^{1.01} \cdot S^{0.306}$

$ML_{100} = 0.878 \cdot W^{0.986} \cdot P^{0.246}$

$MM_{100} = 0.0011 \cdot W^{1.08} \cdot S^{1.11}$

$MM_{100} = 0.404 \cdot W^{1.23} \cdot P^{0.567}$

$DS = 0.00032 \cdot W^{1.17} \cdot S^{0.63} \cdot FTA^{1.10}$

$DS = 0.037 \cdot W^{1.13} \cdot P^{0.53} \cdot FTA^{0.98}$

$FT = 0.00104 \cdot W^{0.65} \cdot S^{1.14} \cdot FTA^{1.22}$

$FT = 1.053 \cdot W^{0.72} \cdot P^{0.71} \cdot FTA^{1.16}$

$QC_{100} = 0.00029 \cdot W^{0.64} \cdot S^{1.35}$

$QC_{100} = 0.0321 \cdot W^{1.08} \cdot P^{0.57}$

where:

E_{100} = cumulative total engineering hours at 100 aircraft (thousands)

T_{100} = cumulative total tooling hour at 100 aircraft (thousands)

ML_{100} = cumulative recurring manufacturing labor hour at 100 aircraft (thousands)

MM_{100} = cumulative recurring materials cost at 100 aircraft (thousands of 1973 dollars)

DS = development support cost (thousands of 1973 dollars)

FT = flight test cost (thousands of 1973 dollars)

QC_{100} = cumulative recurring quality control hours at 100 aircraft (thousands)

W = airframe unit weight (lb)

S = maximum speed (kts)

P = specific power (hp lb)

FTA = number of flight test aircraft

"Cost Estimating Relationships for Tactical Combat Aircraft", which was published by IDA (Institute for Defense Analyses) in 1984, is one of the most current cost models for tactical combat aircraft and is referred to as the IDA model [Ref. 5]. It is based on a sample of twenty-six U.S. military aircraft: fighter, attack, bomber

aircraft, etc. However, seven fighter and attack aircraft are used to develop the CERs for RDT&E (Research, Development, Test and Engineering) cost, and fourteen fighter and attack aircraft for procurement cost.

Table 3 shows CERs used in the IDA model. They are developed to estimate the RDT&E and procurement costs of fighter and attack aircraft. To develop the CERs, overall aircraft characteristics are used, and this is one of the main features of the model. CERs used in the model are based on log-linear regressions. Total production quantity of 400 units is selected as the quantity to obtain the costs for the regression. The major explanatory variables are DCPR (Defense Contractor's Planning Report) weight, thrust DCPR weight, maximum speed at best altitude and IOC (Initial Operational Capability) date. DCPR weight is derived from empty weight by use of the relationships indicated in Table 3.

Costs are provided in two categories: total RDT&E cost and cumulative average flyaway cost. All costs used in the IDA model are in FY 1985 TOA (Total Obligational Authority) dollars. A cumulative average learning curve slope of 0.92 is used to adjust the aircraft cost data [Ref. 5: p.5].

TABLE 3

SELECTED CERS FROM IDA MODEL

$$RD = 2.18 \cdot 10^{-6} \cdot DCPR^{2.0493} \cdot (THRUST\ DCPR)^{1.7} \cdot (1.0239)^{IOC-78}$$

$$FLY = 0.194 \cdot (DCPR\ 1000)^{0.963} \cdot (SP\ 100)^{0.760} \cdot (1.034)^{IOC-78}$$

where:

RD = total RDT&E cost (millions)

FLY = cumulative average flyaway cost of 400 aircraft (millions)

THRUST = total maximum thrust at sea level (lb)

SP = maximum speed at best altitude (kts)

IOC = initial operational capability date (last two digits of calendar year)

DCPR = aircraft Defense Contractor's Planning Report weight (lb)

$DCPR = 0.0913 \cdot (EW)^{1.177}$ for $EW > 50000$

$DCPR = 0.246 \cdot (EW)^{1.096}$ for $10000 \leq EW \leq 50000$

$DCPR = 13.26 \cdot (EW)^{0.674}$ for $EW < 10000$

EW = aircraft empty weight (lb)

Table 4 shows the summarized characteristics of the three models. Since each model has its own purpose, the characteristics are different for each model. However, it is very interesting that the predicted costs from each model are fairly similar. Table 5 compares the predicted F-16 costs for these three models.

TABLE 4

THE CHARACTERISTICS OF THREE MODELS

MODEL	DAPCA-III	Large	IDA
published year	1976	1977	1984
sampled aircraft	several types	fighter only	fighter, attack
sample size	25	17, 31	7, 14
costs of CERs	subsystem	subsystem	overall system
major variables	weight, speed, time of first flight, dummy	weight, speed, specific power	weight, speed, thrust-weight ratio, IOC date
baseline quantity	cumulative 200	cumulative 100	cumulative 400
units of cost	1975 dollars	1973 dollars	1985 dollars

The predicted costs of DAPCA-III and Large models came from summing up all of their subsystem costs. The last page of Large provides a good comparison between the DAPCA-III model and Large model of F-16 cost estimates for 100 aircraft. The estimates range from 8.867 to 10.356 million dollars, with the total flyaway cost by the IDA model being 9.401 million dollars. The actual total flyaway cost of an F-16 for 100 aircraft is 9.641 million dollars according to the "US Military Aircraft Cost Handbook" [Ref. 6: p.IV-337]. So, the predicted cost from the IDA model is a better prediction than the costs given by the other models. There may be several reasons for this result. One of them is that the DAPCA-III and Large models were published earlier than the IDA model. Also, it may be that CERs based on the overall aircraft system are better than CERs based on the subsystems.

TABLE 5

COMPARISON OF PREDICTED F-16 COSTS FOR THREE MODELS

DAPCA-III model		Large model			IDA model	
		fighter only		31-several types aircraft		
with time	without time	with power	with speed		RDT&E cost	flyaway cost
9.839	10.356	10.004	8.867	10.232	1293.082	9.401

note :

1. Costs are based on the total production quantity of 100 units
2. All costs are in constant 1981 dollars (millions)
3. For price-level adjustments, price indices in Appendix B were used
4. Actual cost of an F-16A is 9.641 million dollars [Ref. 6: p.IV-337]

III. DATA COLLECTION AND ADJUSTMENTS

CERs are generally obtained from the statistical analysis of historical data. Data must be collected in order to develop CERs and then adjusted for validity and reliability. Acquisition of data is the process of identifying, searching out, obtaining, verifying, and recording the specific information that is of value to the analyst.

The initial step in developing CERs is identifying the aircraft of interest from the many types of aircraft such as fighters, bombers, cargo carriers, reconnaissance aircraft, helicopters, etc. However, this thesis presents the CERs for fighter aircraft only. The aircraft data used has been collected and adjusted from unclassified sources.

A. DATA COLLECTION

Developing reliable CERs, especially for a military application, is very difficult at best. Consequently there are many problems with the CERs used for military hardware. The most significant problem with data collection on a military system is to obtain complete information from unclassified documents. This has led to data anomalies in weapon system cost estimation.

Early data have not been systematically processed and stored which makes the historical information of little value. In an attempt to alleviate this data collection problem, the Contractor Information Report (CIR) Program was established by the Department of Defense (DOD) in 1966. This reporting system was designed to collect costs and related data on major contracts for aircraft and missile and space programs. The CIR was enlarged to cover the other areas of defense contracting with the implementation of the Contractor Cost Data Reporting System (CCDR). The CCDR collects contractor costs and related data needed to satisfy cost estimating requirements. In recent years, The Analytical Science Corporation (TASC), with the assistance of Management Consulting and Research, Inc. (MCR), has been compiling data and analyzing the cost versus the effectiveness of tactical aircraft produced since 1950.

While collecting data, the levels of accuracy and aggregation should be considered in order to develop new CERs. There are two basic categories of data: aircraft physical and performance parameters and cost. The sample for this thesis consisted of the following aircraft:

F-4E	F-14A	F-86F	F-104C
F-6A	F-15A	F-89D	F-105D
F-8E	F-16A	F-100D	F-106A
F-9F	F-18A	F-101B	F-111A
F-11A	F-84F	F-102A	

The model developed in this thesis is based on this sample of nineteen U.S. fighter aircraft. Since the purpose of this thesis is to provide fighter-based CERs, only fighter aircraft data were collected. The parametric data for fighter aircraft were obtained from references 7 to 11; however, *Jane's All the World's Aircraft* was used primarily. Most of the earlier CERs were out of date in that they did not include aircraft introduced into the armed forces in the 1970's and 1980's, such as the F-14, F-15, F-16 and F-18. However, the data used in this thesis includes the newest fighter aircraft. In order to obtain reliable CERs, all the aircraft included in this thesis had initial flight dates following 1950. Only one aircraft has been selected from each design of fighter aircraft in order to decrease potential multicollinearity in the data sample.

The cost data were obtained from the "US Military Aircraft Cost Handbook" [Ref. 6]. They are based on a cumulative total production quantity of 100 units, so the costs presented in Appendix A are the cumulative average total flyaway costs. All costs used in this thesis are in constant 1981 dollars.

The following definitions were developed and used as a basis for determining what adjustments would have to be made to the data. They are:

1) Weight : maximum take-off gross weight (lb)
2) Thrust : total maximum engine thrust (lb)
3) Speed : maximum speed at best altitude (kts)
4) Year : year of initial operational capability
5) Cost : cumulative Average Costs (CAC) of 100 units for total flyaway cost in constant 1981 dollars (millions)

Like the IDA model, overall aircraft characteristics are used in order to estimate the fighter aircraft costs. The major variables for airframe cost are maximum speed at best altitude, maximum take-off gross weight and initial operational capability year. Some other variables relating to aircraft characteristics (e.g., wing span, maximum thrust, thrust-weight ratio, etc.) were tried and evaluated but generally were found not to be significant. Appendix A shows the total data base used in this thesis.

B. DATA ADJUSTMENT

The distortion of the sample observations used in generating CERs is another significant problem encountered with military hardware. The major distortion occuring is data normalization. Information collected and reported should be adjusted using standardized procedures such as provided by the Cost Accounting Standards Board which establishes consistency in accounting practices among government contractors. Standardization has an important effect upon the ability of DOD contracting personnel to evaluate proposals and better determine allocation and allowability of costs. Additionally, when using data for different purposes, it is necessary to make different adjustments in the data. The two most common adjustments are price-level and cost-quantity adjustments.

1. Price-Level Adjustments

In order to compare the cost of an old system to the cost of a new system, the cost figures must be adjusted to constant dollars. Adjustments are made by means of a price index constructed from a time-series of data in which one year is selected as the base and the value for that year expressed as 100. The other years are then expressed as percentages of this base.

Total Obligational Authority (TOA) dollars in a year (then-year dollars) are the amounts budgeted in a specific fiscal year. The conversion of TOA dollars to constant dollars is accomplished by dividing TOA by a composite index [Ref. 6: p.III-5]. Mathematically, the relationship can be expressed as

$$\text{Constant Dollars} = \frac{\text{TOA}}{\text{composite index}} \times 100$$

Appendix B shows the deflators index and composite indices used by the military services (e.g., Army, Navy, Air Force). The composite indices are based on the Office of the Assistant Secretary of Defense (OASD), Comptroller, deflator for major commodity procurement and service outlay profiles. The tables are based on Fiscal Year (FY) 1981 and all index numbers are related to FY81 constant dollars. So the composite indices are used to normalize aircraft procurement costs of the respective services into FY81 constant dollars. Multiplication by 100 is required since the index is expressed as a percentage.

As an example of price-level adjustment, calculating the total cost of the F-16 is represented. According to the Large model, the total cost of an F-16 from the fighter sample using specific power is 4.84 million in constant 1973 dollars. The composite index of 1973 is 48.38 [Ref. 4: p.15]. Therefore, we can calculate the constant 1981 dollars from the values, that is

$$\text{Constant 1981 Dollars} = \frac{4.84}{48.38} \times 100$$

$$= 10.004 \quad \text{(millions)}$$

2. Cost-Quantity Adjustments

Learning curves, as cost-quantity relationships, are used in order to develop consistent measures of costs. The basis of learning curve theory is that each time the total quantity of items produced doubles, the cost per item is reduced to a constant percentage of its previous cost. So if the average cost of producing all 200 units is 90 percent of the average cost of producing the first 100 units, the process follows a 90 percent cumulative average learning curve.

The cost-quantity relationships are represented using regression analysis techniques assuming the following functional form:

$$C_n = C_1 \cdot n^b \quad \text{or} \quad \ln(C_n) = \ln(C_1) + b \cdot \ln(n)$$

where:

$\ln$ = the natural logarithm function

C_n = cumulative average cost for quantity n

n = cumulative production quantity

C_1 = the cost of the first unit produced

b = the exponent related to the slope of the learning curve

The slope, S, is related to b as

$$S = 2^b \quad \text{or} \quad b = \frac{\ln(S)}{\ln(2)}$$

where:

S = slope expressed as a decimal

Therefore, the coefficient b means that when cumulative production doubles, cumulative average costs decrease by S percent.

As an example of cost-quantity adjustment, calculating the total flyaway cost of the F-6A is represented. The cumulative average cost of 230 aircraft is 3.584 million dollars and 408 aircraft is 3.051 million dollars [Ref. 6: p.IV-278]. So, based upon these two points the learning curve slope can be plotted at about 0.84. As implied earlier, the equation which calculates the cost of n aircraft from the cost of the first unit produced is expressed as

$$C_n = C_1 \cdot n^b$$

where:

$$b = \frac{\ln(S)}{\ln(2)}$$

Therefore,

$$b = \frac{\ln(0.84)}{\ln(2)}$$

$$= -0.25154$$

$$C_{230} = C_1 \cdot 230^{-0.25154}$$

$$= C_1 \cdot (0.25464)$$

Thus,

$$C_1 = \frac{3.584}{0.25464}$$

From this value it is possible to calculate the cumulative average cost of 100 aircraft of the F-6A. The cost is

$$C_{100} = \frac{3.584}{0.25464} \cdot 100^{-0.25154}$$

$$= 4.419 \quad \text{(million dollars)}$$

The costs used in this thesis are Cumulative Average Cost (CAC) for quantity of 100 units. Each fighter has a different learning curve with a unique slope

IV. STATISTICAL APPROACH

CERs are developed from the historical cost of systems and the explanatory variables of those systems. Therefore, some variables which are logically and theoretically related to cost have to be selected in order to develop reliable CERs. An important characteristic of reliable CERs is that the relationship between cost and explanatory variables must be direct and obvious.

Regression analysis can be applied as a statistical technique to develop CERs from the historical cost and parametric data. Regression analysis is primarily concerned with the determination of the equation of a line or curve which will predict how the dependent variable will vary with respect to some independent variables. Therefore, regression analysis will estimate the coefficients of the equation (e.g., intercept and slopes) and infer the reliability and significance of the results of the estimate. (Johnston's *Econometric Methods* [Ref. 12] is the source of all facts and derivations shown in this chapter.)

Generally, there are two types of linear regression models, simple and multiple. The difference between these two models is the number of variables in the equation. The simple linear regression model has only two variables, while the multiple linear regression model has more than two variables.

A. SIMPLE LINEAR REGRESSION

The equation used in simple linear regression has two variables, cost and an explanatory variable. This means that the cost is expressed as a linear function of an explanatory variable. Thus, as an example of the simple linear regression model, the linear relationship is

$$y = \alpha + \beta x + u$$

where:

y = the dependent (cost) variable

x = the independent (explanatory) variable

α = the intercept of the line

β = the slope of the line

u = error term between the actual cost and expected cost of y

Additionally, the log-linear regression model is very frequently used as another method of expressing the linear model. The log-linear equation results from taking logarithms of both sides of the linear equation, and is written as

$$y = e^{\alpha} \cdot x^{\beta} \cdot e^{u} \quad \text{or} \quad \ln(y) = \alpha + \beta \cdot \ln(x) + u$$

Thus, this equation graphs as a linear relationship when plotted in terms of ln(x) and ln(y).

There are some assumptions made with regards to the error term. The first assumption is that the error term is normally distributed with zero mean and variance σ^2, that is

$$u \sim N(0, \sigma^2)$$

The second assumption is that the error term for different x values are independent and identically distributed.

1. Least-Squares Estimation

As implied earlier, the simple linear regression model has some unknown parameters: α, β, and σ^2. Those unknown parameters have to be estimated in order to establish CERs. The least-squares is the most frequently used method for estimating the unknown parameters.

By using the simple linear regression model the actual cost of the system is indicated by

$$y_i = \alpha + \beta x_i + u_i$$

where y_i is the actual cost of the ith observation. Then, any straight line drawn through the scatter of data points may be regarded as an estimate of the hypothesized relationship $y = \alpha + \beta x + u$. A straight line is indicated by

$$y^* = a + bx$$

where y^* indicates the value of the line at any given value of x.

The principle of the least-squares is that the unknown parameters are selected to minimize the sum of squared residuals. This minimization is expressed as

$$\min \Sigma e_i^2$$

Under this principle, the unknown parameters are determined as

$$a = \overline{Y} - b\overline{X}$$

$$b = \frac{\Sigma(x_i - \overline{X})(y_i - \overline{Y})}{\Sigma(x_i - \overline{X})^2}$$

The difference between the actual cost and the expected cost is defined as the residuals which is written as

$$e_i = y_i - y_i^* = y_i - (a + bx_i)$$

where e_i is the residual of the ith observation. Also, under the minimization principle, the unbiased estimator for σ^2 is determined as

$$S^2 = \frac{\Sigma e_i^2}{n-2}$$

The following are some properties of the least-squares. First, the expected values of the parameters a and b are exactly same as the values of α and β. It is indicated by

$$E[a] = \alpha \quad \text{and} \quad E[b] = \beta$$

Thus, a and b, as the least-squares estimators in a simple linear regression model, are unbiased estimators for α and β. Secondly, the least-squares estimators have the minimum variances among all linear unbiased estimators. As a result, the least-squares estimators for a and b are called the best linear unbiased estimators [Ref. 13: p.473]. The minimum variances property is the major reason why least-squares is so frequently employed in estimating unknown parameters.

By using the least-squares, some simple linear regression models are obtained. Then, the log-linear function can be selected as the best simple linear model. An example is

$$C = 0.172 \cdot T^{1.230} \quad \text{or} \quad \ln(C) = \ln(0.172) + 1.230 \cdot \ln(T)$$

and rewrite the model as

$$C' = -1.760 + 1.230 \cdot T'$$

where:

C = total flyaway cost of fighter aircraft in constant 1981 dollars (millions)

T = total maximum engine thrust (lb)

2. The Correlation Coefficient

The selected model must be examined in order to determine the reliability or accuracy of that equation. There are several statistical measures that can indicate the goodness of fit of the equation in describing data. R^2 is the most commonly used measure of the goodness of fit and is defined as the coefficient of determination which comes from squares of the correlation coefficient (R). The computing of R^2 is as follows:

$$R^2 = \frac{\text{Explained sum of squares}}{\text{Total sum of squares}}$$

$$= \frac{\Sigma(y_i^* - \overline{Y})^2}{\Sigma(y - \overline{Y})^2}$$

$$= 1 - \frac{\text{Residual sum of squares}}{\text{Total sum of squares}}$$

$$= 1 - \frac{\Sigma e^2}{\Sigma(y - \overline{Y})^2}$$

R^2 is the proportion of the total deviation which can be explained by the regression model, and corresponds to all data points which lie on the regression line. The highest possible value of R^2 is 1.00 and the lowest is 0.00.

The value of R^2 from the log-linear regression model above is

$$R^2 = 0.7007$$

which is a relatively low value. It means that thrust alone does not explain all of the variance in the cost data. It also means that the log-linear model above does not fit the data well. Usually, there exists two ways to increase R^2 to a relatively high value. They are:

1) To add some other variables into that equation. Adding variables may explain the remaining variance. This will be discussed in Section B below.

2) To find other equations. If the simple linear regression model does not fit the data well, then multiple linear regression models with other variables may fit the data better.

3. **Statistical Inference**

As implied earlier, the hypothesized relationship between the dependent variable (y) and independent variable (x) may be indicated by

$$y = \alpha + \beta x + u$$

where u is an error term. Under this relationship, the least-squares method produces unbiased estimators a and b. Thus, the outcomes of a least-squares regression line is

$$y_i^* = a + bx$$

Standard statistical techniques can be applied to the least-squares result to test for significance and to make inferences about reliability and accuracy in a probabilistic sense.

a. t-test

It is necessary to test the relationship between y and x. This is done by establishing the null hypothesis that y and x are not related to each other, and the alternative hypothesis that y and x are related to each other:

$$H_0: \ \beta = 0$$

$$H_1: \ \beta \neq 0$$

These hypotheses are the most frequently used, and are referred to as testing the significance of x. By a similar development, tests on the intercept are

$$H_0: \ \alpha = 0$$

$$H_1: \ \alpha \neq 0$$

The test that is commonly used for this purpose is known as the t-test because the tests on the α and β are based on the t distribution. It follows then that:

$$t_a = \left| \frac{a - \alpha}{S_a} \right| \sim t(n-2)$$

$$t_b = \left| \frac{b - \beta}{S_b} \right| \sim t(n-2)$$

where:

S_a = the standard error of a

$$= S \cdot \frac{\Sigma x_i^2}{\sqrt{n \cdot \Sigma (x_i - \overline{X})^2}}$$

S_b = the standard error of b

$$= \frac{S}{\sqrt{\Sigma (x_i - \overline{X})^2}}$$

S = the standard error of regression

$$= \frac{\Sigma e^2}{\sqrt{n-2}}$$

If the sample t statistic is numerically greater than the preselected critical value of t, we accept the alternative hypothesis and conclude that x plays a significant role in the determination of y. The following values result from the least-squares regression line based on the data in Appendix A. They are

$a = -1.760444$

$b = 1.230396$

$S_a = 0.586718$

$S_b = 0.195015$

Since $n = 19$, from the t distribution with 17 degrees of freedom,

$$t_{0.025}(17) = 2.110$$

Thus, the intercept is significantly different from zero since

$$t_a = |-3.000| = 3.000 > 2.110$$

Also, the slope is significant since

$$t_b = 6.309 > 2.110$$

b. Confidence Interval

Examining the confidence intervals for α and β is another way to test the significance of the unbiased estimators a and b. Since a confidence interval which includes zero is equivalent to accepting the null hypothesis that the true value of the parameter is zero, an interval which does not include zero is equivalent to rejecting the null hypothesis.

Generally, $100(1-p)$ percent confidence intervals for α and β are indicated by

$$CI(\alpha) = a \pm t_{p/2} \cdot S_a$$

$$CI(\beta) = b \pm t_{p/2} \cdot S_b$$

where S_a and S_b are the standard errors of a and b.

A 95 percent confidence interval for α is then

$$CI(\alpha) = -1.760 \pm (2.110 \times 0.587)$$

or

$$-0.522 \quad \text{to} \quad -2.998$$

Also, a 95 percent confidence interval for β is

$$CI(\beta) = 1.230 \pm (2.110 \times 0.195)$$

or

$$0.189 \quad \text{to} \quad 1.642$$

Therefore, the fact that the confidence intervals for α and β do not include zero means that the null hypotheses are rejected, and the unbiased estimators a and b are statistically significant.

c. F-test

The analysis of variance (ANOVA) test is merely a significance test on β performed in another way, and is referred to as the F-test. The F statistic is the ratio of the mean square due to x over the residual mean square. Thus, it is indicated by

$$F = \frac{b^2 \cdot \Sigma(x - \bar{X})^2}{\Sigma e^2 / (n-2)} \sim F(1, n-2)$$

The significance of x is thus tested by examing whether the sample F exceeds the appropriate critical value of F taken from the upper tail of the F distribution. Therefore, the test procedure is that if the value of F is greater than the value of F(1, n − 2), then reject $H_0 : \beta = 0$.

Usually, the F-test will be applied extensively in multiple linear regression models. However, in simple linear regression models, the F variable with (1,k) degrees of freedom is the square of a t value with k degrees of freedom. The relationship between the t and F distributions can be explained with the correlation coefficient, R^2. It is

$$t = \frac{R \cdot \sqrt{(n-2)}}{\sqrt{1-R^2}}$$

$$F = \frac{R^2 \; 1}{(1-R^2)\,(n-2)} = t^2$$

The ANOVA for the least-squares regression line based on the 19 observations in Appendix A is as follows:

Source	Degrees of freedom	Sum of square	Mean square
Thrust	1	11.238766	11.238766
Residual	17	4.799673	0.282334
Total	18	16.038440	

Since n = 19, using the F distribution with 1 and 17 degrees of freedom,

$$F_{0.95}(1,17) = 4.451$$

The sample F statistic is

$$F = \frac{11.239}{0.282} = 39.807 > 4.451$$

Thus, $H_0 : \beta = 0$ rejected. It means that the intercept is not zero.

B. MULTIPLE LINEAR REGRESSION

In the previous section, the linear relationship between cost and thrust was examined as a simple linear regression model. It was selected as the best model using two variables, and the relationship was represented with log-linear function. However, its low R^2 means that using a model with only one independent variable, thrust, cannot fit the situation well. Therefore, some other models which have more than one independent variables have to be examined.

Multiple linear regression models have more than one independent variable. Thus, the vector of sample observations on the dependent variable (Y), may be expressed as a linear combination of the sample observations on the independent variables (X) and the vector of the error term (u). An example of the hypothesized multiple linear regression model is represented as

$$Y = \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + u$$

where:

Y = the vector for dependent (cost) variable

X_1 = the unit vector for an intercept

X_i = the vector for independent variables ($i \neq 1$)

β_i = unknown parameters

u = the vector of error terms

Each vector is a column vector of n elements. The multiple linear regression model may also be expressed in matrix form as

$$Y = X\beta + u$$

where Y and u are $n \times 1$ matrices, X is a $n \times k$ matrix, and β is a $k \times 1$ matrix.

Like the simple linear regression model, there are some assumptions made for the multiple linear regression model. They are:

1) The u vector has a multivariate normal distribution, with each u distribution having a zero mean vector and the same variance vector (σ^2). That is

$$u \sim N(0, \sigma^2 I)$$

where I is the identity matrix.

2) X is a nonstochastic matrix and its rank is k. That is

$$\rho(X) = k$$

1. Ordinary Least-Squares (OLS) Estimation

As implied earlier, the hypothesized multiple linear regression model and a vector of the straight line are indicated by

$$Y = X\beta + u$$

$$Y^{*} = Xb$$

where b is k element vector. Thus, a vector of errors or residuals can be defined as

$$e = Y - Xb$$

The principle of the least-squares is that b is selected to minimize the sum of the squared residuals, e′e. Under this principle, b is determined as

$$\begin{aligned} b &= (X'X)^{-1}X'Y \\ &= \beta + (X'X)^{-1}X'u \end{aligned}$$

Then the variance-covariance matrix of the OLS estimators is

$$\mathrm{var}(b) = \sigma^2(X'X)^{-1}$$

where the elements on the main diagonal of this matrix give the sampling variances of the corresponding elements of b, and the off-diagonal terms give the sampling covariances.

Since the expected value of b is exactly the same as the value of β, the OLS estimators are linear unbiased estimators. This is indicated by

$$E[b] = \beta$$

Also, since the OLS estimators have the minimum sampling variances among all of the linear unbiased estimators, b is the best linear unbiased estimator (b.l.u.e). Using the OLS, two equations are selected as the best models. They are

$$C_{19} = -701.635 - 0.215W + 0.358Y$$

$$C_{6} = -3994.618 + 0.688W + 2.013Y$$

where:

W = (maximum take-off gross weight) 1000

Y = year of initial operational capability

The former model is based on the 19 data points in Appendix A, while the latter is based on only 6 data points. However, the 6 data points used in the latter model have an initial operational capability year of 1965 or after. It means that the latter model is based on the relatively new aircraft data. Thus, the 6 data points contained in the latter model are

F-4E	F-14A	F-15A
F-16A	F-18A	F-111A

2. **The Correlation Coefficient**

The correlation coefficient is the most commonly used measure of the goodness of fit. Then, the multiple correlation coefficient for the k-variable is defined as

$$R^2 = \frac{\text{Explained sum of squares}}{\text{Total sum of squares}}$$

$$= 1 - \frac{\text{Residual sum of squares}}{\text{Total sum of squares}}$$

$$= 1 - \frac{e'e}{Y'AY}$$

where:

$A = I - (1/n)\iota\iota'$

I = identity matrix

ι = a column vector of n units

$Y'AY$ = the sum of squared deviations in Y

The value of R^2 from the selected multiple regression model based on the 19 observations is

$$R^2_{19} = 0.7504$$

Although this is a slightly higher value than that of the simple regression model, the value is still relatively low. It means that the weight and year do not explain all of the variance in the cost data, and the model does not fit the data well.

However, the value of R^2 from the selected multiple regression model based on the 6 observations is

$$R^2_6 = 0.9441$$

This is a relatively high and good value, thus the weight and year variables fit the 6 data points.

The value of R^2 adjusted for degrees of freedom is useful when comparing different numbers of independent variables, and is referred as the adjusted R^2. The adjusted R^2 is defined as

$$R^2 = 1 - \frac{e'e\,(n-k)}{Y'AY\,(n-1)}$$

Thus, the values of adjusted R^2 for the selected simple and multiple regression models are

$$R^2 = 0.6831$$

$$R^2_{19} = 0.7192$$

$$R^2_6 = 0.9068$$

Therefore, comparing the results of the adjusted R^2 shows that they are almost same as those of R^2.

3. Statistical Inference

The characteristicces of the multiple linear regression models were already mentioned at the beginning of this chapter. According to them, b is indicated by

$$b \sim N(\beta, \sigma^2(X'X)^{-1})$$

Then, the variance of error term, as an estimator of σ^2, is defined as

$$S^2 = \frac{e'e}{n-k}$$

and S is a standard error of the regression.

a. t-test

Since b is the estimated coefficient matrix of X, b_i is the estimated coefficient of X_i in the OLS regression. b is distributed independently of S^2. Thus the t-test of the multiple linear regression is determined as

$$t = \frac{b_i - \beta_i}{S \cdot \sqrt{a_{ii}}} \sim t(n-k)$$

where a_{ii} denotes the ith element on the principal diagonal of $(X'X)^{-1}$.

Hypotheses are established about β, where the null hypothesis is $H_0 : \beta = 0$ and the alternative hypothesis is $H_1 : \beta \neq 0$. Then, the t statistics of the selected multiple linear models are as follows:

Model	Based on 19 obs.	Based on 6 obs.
Intercept	-2.879	-6.185
Weight	3.535	7.088
Year	2.864	6.194

If n = 19, from the t-distribution with 16 degrees of freedom,

$$t_{0.025}(16) = 2.120$$

and if n = 6, with 3 degrees of freedom,

$$t_{0.025}(3) = 3.182$$

Thus, since all of the t statistics based on the selected models are greater than their critical values, the coefficients are not zero.

b. Confidence Interval

The $100(1-p)$ percent confidence intervals for the coefficients of Weight (X_2) and Year (X_3) are indicated by

$$CI(\beta_2) = b_2 \pm t_{p/2} \cdot S_2$$

$$CI(\beta_3) = b_3 \pm t_{p/2} \cdot S_3$$

where S_2 and S_3 are standard errors of b_2 and b_3.

The following values are indicated in the least-squares regression lines based on the data in Appendix A. They are as follows:

Model	Based on 19 obs.	Based on 6 obs.
b2	0.215	0.688
b3	0.358	2.013
S2	0.061	0.097
S3	0.125	0.325

Thus, the 95 percent confidence intervals for β_2 and β_3 based on the 19 observations are

$$CI(\beta_2) = 0.215 \pm (2.120 \times 0.061)$$

or 0.086 to 0.343

$$CI(\beta_3) = 0.358 \pm (2.120 \times 0.125)$$

or 0.093 to 0.623

Also, the confidence intervals based on the 6 observations are

$$CI(\beta_2) = 0.688 \pm (3.182 \times 0.097)$$

or 0.379 to 0.997

$$CI(\beta_3) = 2.013 \pm (3.182 \times 0.325)$$

or 0.979 to 3.047

Therefore, the fact that all of the confidence intervals do not include zero means that b_2 and b_3 based on the 19 and 6 observations are statistically significant.

c. F-test

The t-test is usually used to test the significance of a single coefficient. However, when added to the function of the t-test, the F-test can be used to test the significance of the complete regression and the significance of a subset of coefficients. Thus, the F-test of the multiple linear regression will be a very useful and powerful tool for testing the independent variables, X.

In order to test the elements of β, the linear hypothesis is established as

$$R\beta = r$$

where R is $q \times k$ matrix with rank q, and r is a q element vector. Therefore, if the linear hypothesis is true, the following is obtained

$$(Rb - r) \sim N(0, \sigma^2 R(X'X)^{-1}R')$$

Thus, the F statistic under the linear hypothesis is

$$F = \frac{(Rb-r)'[R(X'X)^{-1}R']^{-1}(Rb-r)\ q}{e'e\ (n-k)}$$

$$\sim F(q, n-k)$$

In order to test the joint significance of Weight (X_2) and Year(X_3), the null hypothesis is established as

$$H_0 : \beta_2 = \beta_3 = 0$$

Then, the F statistic for this hypothesis can be indicated by

$$F = \frac{\text{Explained sum of squares } (k-1)}{\text{Residual sum of squares } (n-k)}$$

$$= \frac{(Y'AY - e'e)\ (k-1)}{e'e\ (n-k)}$$

Thus, the F statistic based on the 19 observations is

$$F = \frac{810.264\ (3-1)}{269.490\ (19-3)}$$

$$= 24.053$$

Since $n = 19$, from the F distribution with 2 and 16 degrees of freedom,

$$F_{0.95}(2,16) = 3.634 < 24.053$$

Therefore, $H_0 : \beta_2 = \beta_3 = 0$ is rejected. It means that even though the sample R^2 is numerically low, the model is significant.

Also, the F statistic based on the 6 observations is

$$F = \frac{300.204\ (3-1)}{17.773\ (6-3)}$$

$$= 25.337$$

Since n = 6, from the F distribution with 2 and 3 degrees of freedom,

$$F_{0.95}(2,3) = 9.552 < 25.337$$

Thus, $H_0 : \beta_2 = \beta_3 = 0$ is also rejected. This model is therefore significant with a numerically high R^2.

V. ANALYSIS OF THE MODELS

The reliable CERs will accurately predict the costs of systems, provided they are suitable for that particular system. Thus, in order to establish reliable CERs, the previous chapter demonstrated use of regression methods on the simple and multiple linear regression models performed on various combination of the explanatory variables contained in Appendix A. Then, some models were selected as desirable for predicting costs of fighter aircraft using least-squares estimation. However, many alternative models were discarded because of statistical problems. Appendix C illustrates use of simple and multiple linear regression models for various combinations of the explanatory variables.

For selecting reliable models, approximately 1000 models were estimated. Models were evaluated using from one to eight explanatory variables. The summary of these models is presented below:

	19 observations	6 observations
1 ~ 3 variables	See Appendix C	
4 ~ 5 variables	Statistically unsatisfactory	
6 ~ 8 variables	Statistically unsatisfactory	No report

Then, in order to check how the models fit the data, the selected models were evaluated with several statistical measures: the coefficient of determination (R^2), the adjusted coefficient of determination (R^2), standard error (SE), t statistics (t), confidence intervals (CI), and F statistics (F). However, since no single statistic can be a meaningful indication of the models' applicability, the models' statistics must be looked at together. Table 6 shows a summary of the cost estimating models developed. The table includes the selected equations, the results of the statistical measures, and the correlation matrices of the estimated coefficients in order to aid in analyzing the models.

TABLE 6

SUMMARY OF COST ESTIMATING MODELS

A. Simple linear regression model based on 19 observations

$\ln(C) = \ln(0.172) + 1.230 \cdot \ln(T)$

$R^2 = 0.7007$ $\quad R^2 = 0.6831$ $\quad SE = 0.531$

$t(b1) = -3.000$ $\quad t(b2) = 6.309$

$CI(b2) = 0.189$ to 1.642

$F = 39.807$

CORRELATION MATRIX OF ESTIMATES

	INTERCEP	LPWR
INTERCEP	1.0000	-0.9782
LPWR	-0.9782	1.0000

B. Multiple linear regression model based on 19 observations

$C = -701.635 + 0.215W + 0.358Y$

$R^2 = 0.7504$ $\quad R^2 = 0.7192$ $\quad SE = 4.104$

$t(b1) = -2.879$ $\quad t(b2) = 3.535$ $\quad t(b3) = 2.864$

$CI(b2) = 0.086$ to 0.343 $\quad CI(b3) = 0.093$ to 0.623

$F = 24.053$

CORRELATION MATRIX OF ESTIMATES

	INTERCEP	WT	YEAR
INTERCEP	1.0000	0.5662	-1.0000
WT	0.5662	1.0000	-0.5731
YEAR	-1.0000	-0.5731	1.0000

C. Multiple linear regression model based on 6 observations

$C = -3994.618 + 0.688W + 2.013Y$

$R^2 = 0.9441$ $\quad R^2 = 0.9068$ $\quad SE = 2.434$

$t(b1) = -6.185$ $\quad t(b2) = 7.088$ $\quad t(b3) = 6.194$

$CI(b2) = 0.379$ to 0.997 $\quad CI(b3) = 0.979$ to 3.047

$F = 25.337$

CORRELATION MATRIX OF ESTIMATES

	INTERCEP	WT	YEAR
INTERCEP	1.0000	-0.8239	-1.0000
WT	-0.8239	1.0000	0.8209
YEAR	-1.0000	0.8209	1.0000

Additionally, characteristics other than statistical measures should be considered in analyzing the models. Some of them are:

1) The signs and the magnitudes. Usually, cost is expected to increase with thrust and weight. Additionally, since the new aircraft contain modular avionics which are easily updated (e.g., radar, electronic equipments, etc.), the cost of the new aircraft is expected to increase with year of initial operational capability. Therefore, the developed models containing the positive coefficients for thrust, weight, and year make sense.

2) The constant term. The developed multiple linear regression models contained large negative constant terms. This means that the developed multiple linear regression models would not be valid over the full range of possible values of the independent variables.

3) The correlation matrix. The correlation matrices are included in the table to aid in determining the multicollinearities that may exist between the various independent variables in the models.

Table 6 shows that all of the t statistics are greater than their critical values, and the confidence intervals do not include zero. This means that all of the unbiased estimators of the developed models are significantly different from zero. Furthermore, since all of the F statistics are greater than their critical values, the developed models are significant.

However, the multiple linear regression model based on 6 observations which has an initial operational capability year following 1965 contains desirable values of the coefficient of determination (R^2), 0.9441, and the coefficient of determination adjusted for degrees of freedom (R^2), 0.9068. This indicates that the equation based on 6 observations fits the data well because the dependent variables, weight and year, explain the variance in the cost data.

Also, Table 6 shows that the multiple linear regression models based on 19 observations contains a large value of standard error (SE) which is a measure of the dispersion of the data and relates to the prediction intervals. It indicates that the multiple linear regression model based on 19 observations does not have the desirable prediction intervals. Therefore, the multiple linear regression model based on 6 observations is selected as a desirable fighter aircraft CER.

Initially, the data base contained a large number of international fighter aircraft, but many observations were eliminated because of insufficient information. As such, only 19 observations were chosen. Since models using 19 observations were statistically unsatisfactory, a small subset of 6 observations was selected from the original 19. Then, the Chow test [Ref. 12: p.207-225] was performed on models with 6 observations and related with the other 13 observations (i.e., comparisons were made to determine if both data sets came from the same population of fighter aircraft). Appendix D shows the test results which indicates that the two groups of data are not from the same population. The 6 observations are representative of current fighter aircraft, and should provide the best estimates of future fighter aircraft costs.

Since the purpose of CERs is to estimate the cost of systems, by substituting the parameters of the proposed system into the CERs, it will be possible to estimate the cost of the system. There are two kinds of prediction: a point prediction and an interval prediction. If the obtained equation fits the data well, then a good prediction will be possible. However, it is very unlikely that the point prediction will be realized. Therefore, a prediction interval should be constructed in order to describe the uncertainty of the estimates.

Point prediction is obtained by substituting the values of dependent variables into the selected equation. As implied earlier, the selected multiple linear regression model based on 6 observations is

$$C_6 = -3994.618 + 0.688W + 2.013Y$$

Thus, since the value of weight (W) and year (Y) of the F-16A are 35.4 and 1978, the selected regression equation gives the point estimate of an F-16A as follows:

$$\begin{aligned} C_6 &= -3994.618 + 0.688(35.4) + 2.013(1978) \\ &= 11.917 \end{aligned}$$

Also, the following formula is used to construct a $100(1-p)$ percent prediction interval (PI) for the point estimate. It includes the standard error (SE) and indicates as follows:

$$PI = Y_f^* \pm t_{p/2} \cdot SE \cdot \sqrt{1 + R(X'X)^{-1}R'}$$

where Y_f^* is the point forecast, X is the matrix of data base with the first column of units, and R is the vector of proposed system's parameters.

Therefore, a 95 percent prediction interval for F-16A based on 6 observations is

$$Y_f^* = 11.917$$

$$t_{0.025}(3) = 3.182$$

$$SE = 2.434$$

$$(X'X)^{-1} = \begin{pmatrix} 70402.8289 & -8.7208 & -35.4297 \\ -8.7208 & 0.0016 & 0.0044 \\ -35.4297 & 0.0044 & 0.0178 \end{pmatrix}$$

$$R(X'X)^{-1}R' = 0.519$$

Thus,

$$\begin{aligned} PI &= 11.917 \pm 2.120 \cdot (2.434) \cdot \sqrt{1.519} \\ &= 11.917 \pm 6.360 \end{aligned}$$

or 5.557 to 18.277

Up to this point we have seen some reasons to believe that the multiple linear regression model based on 6 observations will give a better estimate of fighter aircraft than more broadly based models. However, in order to aid in comparing the selected models, Table 7 shows a summary of the cost predictions. It includes the cost predictions of F-16A and F-18A.

As a result, the table verifies that the multiple linear regression model based on 6 observations gives a better estimate than those of the other models. This means that, since the 6 observations are new fighter aircraft with a initial operational capability year following 1965, a model based on new aircraft data may correctly predict the cost of a new fighter aircraft.

The cost used in this thesis is cumulative average costs of 100 units for total flyaway cost in 1981 millions of dollars.

TABLE 7

SUMMARY OF COST PREDICTIONS

		F-16A	F-18A
Actual cost		9.641	23.968
Point Prediction	S_{19}	9.025	12.231
	M_{19}	14.527	17.853
	M_6	11.917	23.444
Prediction Intervals	S_{19}	2.841 ~ 28.691	3.800 ~ 39.370
	M_{19}	4.145 ~ 24.909	7.817 ~ 27.889
	M_6	5.557 ~ 18.277	13.860 ~ 33.028

where:

S_{19} = simple linear regression model based on 19 observations

M_{19} = multiple linear regression model based on 19 observations

M_6 = multiple linear regression model based on 6 observations

VI. CONCLUSION

This thesis presented a regression model of a CER for fighter aircraft. It is based on 19 fighter aircraft because the major objective of this thesis is developing CERs for fighter aircraft only.

As implied earlier, there are many CERs for aircraft. They are very useful for developing new CERs but are different from each other. The differences mostly depend upon the aircraft types, the included aircraft data, and the statistical methods used. However, even though they are different from each other, their results are similar. This means that since the purpose of CERs is to provide a reasonable cost estimation of systems, they give similar estimates of a particular aircraft.

As a result of this thesis, a multiple linear regression model based on 6 observations is selected as the best model to estimate the costs of fighter aircraft. Then, it is a very meaningful result because the 6 observations are new fighter aircraft with an initial operational capability year following 1965. There may be several reasons for this result such as the limited data base of the model or the applied statistical methods. But the most reasonable cause of the result is the characteristics of the data. Traditionally, every new fighter aircraft requires large development costs. Also, it includes developed systems such as radar, electronic equipments, armament systems, etc. Undoubtedly, those systems are very expensive. However, those characteristics usually were not considered as the explanatory variables. This means that a model based on old technology may incorrectly estimate the cost of a new system containing advanced technology. Therefore, in order to estimate the costs of modern or future fighter aircraft, CERs should be based on new aircraft data.

There were some difficulties in developing CERs for fighter aircraft. The data problem was the first and most difficult problem. Sufficient numbers of observations can support the distribution assumptions and reduce the standard error. Thus, CERs based on sufficient numbers of observations may give better confidence or prediction intervals because these are functions of the standard error. However, since the fighter aircraft data used in this thesis were very limited, it caused quite a wide standard error and wide confidence or prediction intervals.

Similarly, accuracy of the data is very important. Inaccurate data is worthless because it cannot lead to reliable CERs. Thus, under such conditions, it is very hard to expect accurate estimates. However, some explanatory variables of new fighter aircraft data were classified such as the maximum speed of F-15A. But, the selected models were not very good CERs.

Additionally, as a statistical method, OLS has some problems. OLS is almost exclusively the selected regression technique. It is based on the assumptions that the error term is normally distributed, and the estimates are selected to minimize the sum of the squared deviations of actual cost observations from their estimates. However, OLS as a regression method is quite sensitive to outlying observations. If the data base includes some unusual observations then it tends to give a poor result. Thus, there is a tendency to discard those observations that seem to lie outside a normal trend line in order to remove a possible bias in the estimating equation.

Finally, further study and developments of CERS for fighter aircraft should consider the following:

1) Use accurate and sufficient data. The purpose of this study is to get reliable CERs which gives an accurate cost estimate of the systems. This is possible by using accurate data. Furthermore, sufficient data can reduce the standard error so that it gives accurate confidence and prediction intervals, because they depend upon the standard error.

2) Use alternate methods. OLS is the most frequently used estimating technique for CERs, but it is not a perfect technique by itself. Thus, it is needed to support and compare the established CERs, but alternate methods, such as generalized least squares or least absolute value regression, will also do that.

Additionally, in order to estimate the costs of modern or future systems, it is important also to suggest that the new data should be added to the model and old ones removed. That enables the model to be kept updated and restricted to fighter aircraft with similar characteristics.

APPENDIX A
AIRCRAFT DATA

A/C	Cost	Span	Thrust	Weight	Speed	SER	Year
F4E	5.919	38.6	35.80	61.795	1394	1	1966
F6A	4.419	33.5	14.50	25.000	612	2	1952
F8E	3.297	35.2	18.00	34.000	986	2	1961
F9F	.930	38.0	5.75	16.450	463	2	1951
F11A	4.895	31.6	10.50	24.078	783	2	1953
F14A	23.901	64.1	41.80	74.349	1342	2	1971
F15A	19.356	42.8	50.00	56.000	1440	1	1973
F16A	9.641	31.0	25.00	35.400	1150	1	1978
F18A	23.968	37.5	32.00	49.224	980	2	1979
F84F	5.943	33.6	7.22	28.000	579	1	1951
F86F	1.095	39.1	5.91	20.611	537	1	1951
F89D	3.496	59.7	14.40	41.000	537	1	1951
F100D	2.659	38.8	16.95	34.832	760	1	1954
F101B	7.419	39.7	29.98	46.673	1074	1	1956
F102A	9.206	38.1	17.20	31.500	726	1	1953
F104C	4.612	21.9	15.80	23.590	1276	1	1956
F105D	10.637	34.9	26.50	52.546	1223	1	1958
F106A	11.255	38.3	24.50	38.250	1342	1	1957
F111A	23.510	63.0	37.00	91.500	1452	1	1965

note :

A/C = type of fighter aircraft

Cost = cumulative Average Costs (CAC) of 100 units for total flyaway cost in constant 1981 dollars (millions)

Span = wing span (ft)

Thrust = (total maximum engine thrust)/1000 (lb)

Weight = (maximum take-off gross weight)/1000 (lb)

Speed = maximum speed at best altitude (kts)

SER = identification of service (Navy = 2, Air Force = 1)

Year = year of initial operational capability

Source : References 6 - 11

APPENDIX B
PRICE INDEX

	DOD OUTLAY	COMPOSITE INDEXES		
FISCAL YEAR	ESCALATION INDEX	APN	APA	APAF
1950	26.44	28.68	29.00	28.80
1951	29.03	29.13	29.11	29.19
1952	29.15	29.12	29.24	29.09
1953	28.91	29.46	30.09	29.80
1954	28.45	30.53	31.29	30.80
1955	30.34	31.79	32.64	32.17
1956	30.93	33.11	33.34	33.21
1957	33.62	33.36	33.10	33.28
1958	33.60	32.92	32.76	32.84
1959	33.06	32.69	32.77	32.68
1960	32.39	32.84	32.83	32.89
1961	33.00	32.79	32.63	32.68
1962	33.01	32.62	32.82	32.74
1963	31.93	33.06	33.68	33.30
1964	32.54	34.21	35.40	34.72
1965	32.88	36.25	37.57	36.81
1966	35.47	38.40	39.46	38.87
1967	37.75	40.13	40.98	40.51
1968	39.67	41.57	42.57	42.00
1969	40.73	43.30	44.51	43.82
1970	42.32	45.39	46.70	45.97
1971	44.28	47.70	49.33	48.41
1972	46.14	50.61	53.09	51.68
1973	48.38	57.41	58.02	56.17
1974	52.47	60.69	62.86	61.32
1975	58.44	64.83	66.98	66.06
1976	63.79	69.53	71.10	70.84
1977	68.67	77.60	78.73	79.00
1978	73.57	85.45	87.05	86.77
1979	80.16	96.49	96.53	97.33
1980	89.62	106.54	107.62	106.30
1981	100.00	117.62	119.60	118.20
1982	114.30	126.36	128.40	126.32
1983	121.73	134.62	136.34	134.64
1984	130.12	143.06	144.80	143.08
1985	138.41	151.68	153.45	151.69
1986	146.87	160.60	162.45	160.59
1987	155.46	169.99	171.95	169.99
1988	164.55	179.94	182.01	179.93
1989	174.18	190.47	192.66	190.46

Source : Reference 6

APPENDIX C
SIMPLE AND MULTIPLE LINEAR REGRESSION MODELS

1) Models with 3 variables

VAR'	DATA	FUNCTION	R^2	PROB > F	PROB > $\lvert t_{b1} \rvert$	PROB > $\lvert t_{b2} \rvert$	PROB > $\lvert t_{b3} \rvert$	PROB > $\lvert t_{b4} \rvert$
span thrust weight	19 obs.	linear	0.69	0.01	0.29	0.73	0.09	0.46
		loglinear	0.73	0.01	0.38	0.45	0.24	0.25
	6 obs.	linear	0.50	0.64	0.95	0.43	0.85	0.64
		loglinear	0.54	0.60	0.66	0.34	0.80	0.47
span thrust speed	19 obs.	linear	0.68	0.01	0.33	0.27	0.04	0.99
		loglinear	0.70	0.01	0.52	0.77	0.06	0.76
	6 obs.	linear	0.87	0.19	0.21	0.10	0.22	0.12
		loglinear	0.77	0.32	0.25	0.20	0.41	0.20
span thrust ser	19 obs.	linear	0.71	0.01	0.10	0.23	0.01	0.23
		loglinear	0.70	0.01	0.30	0.89	0.01	0.76
	6 obs.	linear	0.65	0.47	0.88	0.47	0.73	0.37
		loglinear	0.55	0.59	0.73	0.56	0.84	0.46
span thrust year	19 obs.	linear	0.75	0.01	0.06	0.07	0.15	0.06
		loglinear	0.72	0.01	0.40	0.78	0.01	0.41
	6 obs.	linear	0.80	0.28	0.19	0.13	0.64	0.19
		loglinear	0.85	0.21	0.12	0.11	0.68	0.12
span thrust t/w	19 obs.	linear	0.68	0.01	0.81	0.63	0.02	0.68
		loglinear	0.73	0.01	0.38	0.45	0.01	0.25
	6 obs.	linear	0.53	0.61	0.60	0.39	0.71	0.58
		loglinear	0.54	0.59	0.65	0.34	0.59	0.47
span weight speed	19 obs.	linear	0.63	0.01	0.55	0.99	0.15	0.67
		loglinear	0.70	0.01	0.53	0.41	0.05	0.67
	6 obs.	linear	0.67	0.44	0.42	0.40	0.93	0.40
		loglinear	0.69	0.42	0.47	0.30	0.67	0.41
span weight ser	19 obs.	linear	0.68	0.01	0.51	0.50	0.01	0.15
		loglinear	0.72	0.01	0.47	0.06	0.01	0.32
	6 obs.	linear	0.62	0.50	0.91	0.69	0.95	0.49
		loglinear	0.59	0.54	0.77	0.47	0.64	0.61

span weight year	19 obs.	linear	0.76	0.01	0.01	0.50	0.10	0.01
		loglinear	0.73	0.01	0.19	0.23	0.01	0.19
	6 obs.	linear	0.95	0.08	0.05	0.79	0.12	0.05
		loglinear	0.92	0.11	0.08	0.73	0.27	0.08
span weight t/w	19 obs.	linear	0.69	0.01	0.11	0.60	0.02	0.11
		loglinear	0.73	0.01	0.38	0.45	0.01	0.24
	6 obs.	linear	0.63	0.63	0.84	0.43	0.76	0.80
		loglinear	0.54	0.59	0.65	0.34	0.59	0.79
span speed ser	19 obs.	linear	0.63	0.01	0.01	0.02	0.01	0.16
		loglinear	0.53	0.01	0.00	0.12	0.01	0.60
	6 obs.	linear	0.67	0.44	0.62	0.37	0.63	0.90
		loglinear	0.66	0.46	0.51	0.31	0.48	0.85
span speed year	19 obs.	linear	0.74	0.01	0.01	0.01	0.22	0.01
		loglinear	0.67	0.01	0.13	0.16	0.02	0.14
	6 obs.	linear	0.78	0.31	0.44	0.12	0.85	0.43
		loglinear	0.83	0.23	0.27	0.08	0.93	0.27
span speed t/w	19 obs.	linear	0.61	0.01	0.01	0.01	0.14	0.25
		loglinear	0.64	0.01	0.01	0.08	0.03	0.39
	6 obs.	linear	0.83	0.24	0.80	0.09	0.17	0.29
		loglinear	0.81	0.27	0.27	0.10	0.19	0.32
span ser year	19 obs.	linear	0.71	0.01	0.01	0.01	0.90	0.01
		loglinear	0.55	0.01	0.01	0.28	0.42	0.01
	6 obs.	linear	0.77	0.32	0.37	0.20	0.90	0.36
		loglinear	0.85	0.21	0.17	0.11	0.69	0.17
span ser t/w	19 obs.	linear	0.57	0.01	0.01	0.01	0.51	0.01
		loglinear	0.49	0.01	0.29	0.03	0.66	0.01
	6 obs.	linear	0.68	0.43	0.67	0.30	0.38	0.61
		loglinear	0.59	0.54	0.62	0.36	0.49	0.64
span year t/w	19 obs.	linear	0.72	0.01	0.01	0.01	0.01	0.49
		loglinear	0.59	0.01	0.07	0.11	0.08	0.15
	6 obs.	linear	0.77	0.32	0.24	0.13	0.25	0.90
		loglinear	0.83	0.23	0.16	0.10	0.16	0.96
thrust weight speed	19 obs.	linear	0.70	0.01	0.58	0.09	0.16	0.56
		loglinear	0.72	0.01	0.33	0.24	0.36	0.74
	6 obs.	linear	0.82	0.25	0.19	0.19	0.14	0.13
		loglinear	0.58	0.56	0.35	0.44	0.44	0.31

thrust weight ser	19 obs.	linear	0.73	0.01	0.06	0.07	0.11	0.15
		loglinear	0.72	0.01	0.04	0.06	0.34	0.63
	6 obs.	linear	0.63	0.50	0.89	0.68	0.52	0.30
		loglinear	0.47	0.67	0.82	0.74	0.77	0.41
thrust weight year	19 obs.	linear	0.75	0.01	0.07	0.79	0.06	0.07
		loglinear	0.73	0.01	0.34	0.23	0.31	0.34
	6 obs.	linear	0.99	0.02	0.01	0.11	0.01	0.01
		loglinear	0.92	0.12	0.05	0.80	0.06	0.05
thrust weight t/w	19 obs.	linear	0.69	0.01	0.68	0.55	0.48	0.97
		loglinear	0.72	0.01	0.04	0.05	0.37	.
	6 obs.	linear	0.29	0.84	0.86	0.91	0.78	0.85
		loglinear	0.20	0.71	0.80	0.73	0.72	.
thrust speed ser	19 obs.	linear	0.68	0.01	0.55	0.01	0.85	0.26
		loglinear	0.70	0.01	0.48	0.02	0.80	0.72
	6 obs.	linear	0.51	0.63	0.98	0.67	0.99	0.48
		loglinear	0.46	0.69	0.88	0.61	0.85	0.65
thrust speed year	19 obs.	linear	0.69	0.01	0.19	0.05	0.59	0.19
		loglinear	0.71	0.01	0.43	0.05	0.89	0.43
	6 obs.	linear	0.40	0.75	0.69	0.37	0.51	0.69
		loglinear	0.38	0.38	0.96	0.39	0.64	0.97
thrust speed t/w	19 obs.	linear	0.68	0.01	0.55	0.01	0.79	0.27
		loglinear	0.72	0.01	0.33	0.01	0.74	0.36
	6 obs.	linear	0.71	0.40	0.20	0.16	0.22	0.25
		loglinear	0.58	0.56	0.35	0.25	0.31	0.44
thrust ser year	19 obs.	linear	0.70	0.01	0.31	0.02	0.37	0.31
		loglinear	0.71	0.01	0.46	0.01	0.95	0.46
	6 obs.	linear	0.51	0.63	0.98	0.54	0.37	0.98
		loglinear	0.46	0.68	0.81	0.52	0.50	0.81
thrust ser t/w	19 obs.	linear	0.72	0.01	0.79	0.01	0.19	0.20
		loglinear	0.72	0.01	0.04	0.01	0.63	0.34
	6 obs.	linear	0.57	0.56	0.76	0.44	0.34	0.65
		loglinear	0.47	0.67	0.82	0.52	0.41	0.77
thrust year t/w	19 obs.	linear	0.74	0.01	0.08	0.01	0.08	0.10
		loglinear	0.73	0.01	0.34	0.01	0.34	0.31
	6 obs.	linear	0.91	0.13	0.06	0.05	0.06	0.06
		loglinear	0.92	0.12	0.05	0.05	0.05	0.06

weight speed ser	19 obs.	linear	0.68	0.01	0.06	0.01	0.35	0.13
		loglinear	0.70	0.01	0.01	0.02	0.11	0.42
	6 obs.	linear	0.60	0.53	0.78	0.49	0.83	0.54
		loglinear	0.45	0.69	0.85	0.62	0.85	0.68
weight speed year	19 obs.	linear	0.75	0.01	0.01	0.01	0.65	0.02
		loglinear	0.72	0.01	0.19	0.03	0.33	0.19
	6 obs.	linear	0.96	0.06	0.04	0.02	0.49	0.04
		loglinear	0.95	0.08	0.04	0.03	0.37	0.04
weight speed t/w	19 obs.	linear	0.70	0.01	0.04	0.01	0.41	0.09
		loglinear	0.72	0.01	0.33	0.01	0.74	0.24
	6 obs.	linear	0.92	0.12	0.97	0.04	0.06	0.08
		loglinear	0.58	0.56	0.35	0.25	0.31	0.44
weight ser year	19 obs.	linear	0.77	0.01	0.02	0.01	0.33	0.02
		loglinear	0.71	0.01	0.10	0.01	0.92	0.10
	6 obs.	linear	0.94	0.08	0.07	0.05	0.94	0.07
		loglinear	0.99	0.01	0.01	0.01	0.03	0.01
weight ser t/w	19 obs.	linear	0.73	0.01	0.02	0.01	0.14	0.09
		loglinear	0.72	0.01	0.04	0.01	0.63	0.06
	6 obs.	linear	0.69	0.42	0.57	0.29	0.24	0.50
		loglinear	0.47	0.67	0.82	0.52	0.41	0.74
weight year t/w	19 obs.	linear	0.75	0.01	0.06	0.01	0.06	0.87
		loglinear	0.73	0.01	0.34	0.01	0.34	0.23
	6 obs.	linear	0.99	0.02	0.01	0.01	0.01	0.13
		loglinear	0.92	0.12	0.05	0.05	0.05	0.79
speed ser year	19 obs.	linear	0.61	0.01	0.04	0.19	0.54	0.04
		loglinear	0.62	0.01	0.12	0.05	0.99	0.13
	6 obs.	linear	0.48	0.66	0.79	0.61	0.30	0.79
		loglinear	0.41	0.73	0.72	0.61	0.37	0.72
speed ser t/w	19 obs.	linear	0.47	0.02	0.23	0.01	0.20	0.84
		loglinear	0.56	0.01	0.04	0.01	0.63	0.92
	6 obs.	linear	0.46	0.68	0.92	0.63	0.34	0.96
		loglinear	0.36	0.78	0.82	0.74	0.42	0.93
speed year t/w	19 obs.	linear	0.62	0.01	0.01	0.13	0.01	0.29
		loglinear	0.63	0.01	0.10	0.05	0.11	0.81
	6 obs.	linear	0.26	0.87	0.52	0.57	0.52	0.49
		loglinear	0.33	0.81	0.45	0.52	0.45	0.44

2) Models with 2 variables

VAR'	DATA	FUNCTION	R^2	PROB > F	PROB > $\lvert t_{b1} \rvert$	PROB > $\lvert t_{b2} \rvert$	PROB > $\lvert t_{b3} \rvert$
span	19 obs.	linear	0.68	0.01	0.14	0.24	0.01
		loglinear	0.70	0.01	0.29	0.87	0.01
thrust	6 obs.	linear	0.43	0.43	0.92	0.31	0.84
		loglinear	0.37	0.50	0.66	0.39	0.92
span	19 obs.	linear	0.63	0.01	0.65	0.66	0.01
		loglinear	0.70	0.01	0.48	0.09	0.01
weight	6 obs.	linear	0.49	0.36	0.91	0.30	0.56
		loglinear	0.52	0.33	0.65	0.23	0.39
span	19 obs.	linear	0.58	0.01	0.02	0.02	0.01
		loglinear	0.62	0.01	0.01	0.11	0.01
speed	6 obs.	linear	0.67	0.19	0.27	0.09	0.23
		loglinear	0.65	0.21	0.28	0.10	0.22
span	19 obs.	linear	0.27	0.08	0.37	0.03	0.69
		loglinear	0.10	0.42	0.49	0.21	0.75
ser	6 obs.	linear	0.62	0.23	0.89	0.25	0.29
		loglinear	0.54	0.32	0.73	0.31	0.37
span	19 obs.	linear	0.71	0.01	0.01	0.01	0.01
		loglinear	0.52	0.01	0.01	0.27	0.01
year	6 obs.	linear	0.77	0.11	0.12	0.05	0.12
		loglinear	0.83	0.07	0.06	0.03	0.06
span	19 obs.	linear	0.55	0.01	0.01	0.01	0.01
		loglinear	0.49	0.01	0.27	0.03	0.01
t/w	6 obs.	linear	0.48	0.37	0.63	0.20	0.58
		loglinear	0.45	0.40	0.46	0.22	0.54
thrust	19 obs.	linear	0.69	0.01	0.16	0.08	0.16
		loglinear	0.72	0.01	0.04	0.05	0.37
weight	6 obs.	linear	0.63	0.63	0.98	0.72	0.53
		loglinear	0.20	0.72	0.81	0.74	0.72
thrust	19 obs.	linear	0.65	0.01	0.92	0.01	0.61
		loglinear	0.70	0.01	0.51	0.01	0.85
speed	6 obs.	linear	0.34	0.54	0.45	0.31	0.43
		loglinear	0.38	0.49	0.42	0.28	0.37

thrust	19 obs.	linear	0.68	0.01	0.22	0.01	0.21
		loglinear	0.70	0.01	0.01	0.01	0.75
ser	6 obs.	linear	0.51	0.34	0.94	0.41	0.23
		loglinear	0.44	0.41	0.85	0.45	0.30
thrust	19 obs.	linear	0.69	0.01	0.18	0.02	0.18
		loglinear	0.71	0.01	0.41	0.01	0.41
year	6 obs.	linear	0.21	0.70	0.66	0.45	0.66
		loglinear	0.29	0.59	0.50	0.36	0.50
thrust	19 obs.	linear	0.68	0.01	0.58	0.01	0.22
		loglinear	0.72	0.01	0.04	0.01	0.37
t/w	6 obs.	linear	0.25	0.65	0.63	0.42	0.57
		loglinear	0.20	0.72	0.81	0.48	0.72
weight	19 obs.	linear	0.63	0.01	0.17	0.01	0.53
		loglinear	0.69	0.01	0.01	0.02	0.12
speed	6 obs.	linear	0.49	0.36	0.27	0.19	0.30
		loglinear	0.39	0.48	0.38	0.27	0.37
weight	19 obs.	linear	0.66	0.01	0.08	0.01	0.17
		loglinear	0.65	0.01	0.01	0.01	0.54
ser	6 obs.	linear	0.58	0.27	0.78	0.30	0.21
		loglinear	0.44	0.42	0.98	0.46	0.31
weight	19 obs.	linear	0.75	0.01	0.01	0.01	0.01
		loglinear	0.71	0.01	0.07	0.01	0.07
year	6 obs.	linear	0.94	0.01	0.01	0.01	0.01
		loglinear	0.92	0.02	0.01	0.01	0.01
weight	19 obs.	linear	0.68	0.01	0.04	0.01	0.10
		loglinear	0.72	0.01	0.04	0.01	0.05
t/w	6 obs.	linear	0.28	0.61	0.81	0.39	0.68
		loglinear	0.20	0.72	0.81	0.48	0.74
speed	19 obs.	linear	0.47	0.01	0.13	0.01	0.19
		loglinear	0.56	0.01	0.01	0.01	0.59
ser	6 obs.	linear	0.46	0.40	0.84	0.52	0.21
		loglinear	0.36	0.51	0.75	0.64	0.29
speed	19 obs.	linear	0.59	0.01	0.01	0.23	0.01
		loglinear	0.62	0.01	0.09	0.03	0.09
year	6 obs.	linear	0.08	0.99	0.93	0.99	0.93
		loglinear	0.04	0.95	0.82	0.92	0.82

3) Models with 1 variable

VAR'	DATA	FUNCTION	R^2	PROB > F	PROB > $\|t_{b1}\|$	PROB > $\|t_{b2}\|$
span	19 obs.	linear	0.27	0.02	0.39	0.02
		loglinear	0.10	0.20	0.47	0.20
	6 obs.	linear	0.42	0.17	0.95	0.17
		loglinear	0.37	0.20	0.58	0.20
thrust	19 obs.	linear	0.65	0.01	0.43	0.01
		loglinear	0.70	0.01	0.01	0.01
	6 obs.	linear	0.15	0.45	0.80	0.45
		loglinear	0.16	0.44	0.85	0.44
weight	19 obs.	linear	0.62	0.01	0.19	0.01
		loglinear	0.64	0.01	0.01	0.01
	6 obs.	linear	0.23	0.34	0.64	0.34
		loglinear	0.16	0.43	0.96	0.43
speed	19 obs.	linear	0.41	0.01	0.29	0.01
		loglinear	0.55	0.01	0.01	0.01
	6 obs.	linear	0.01	0.89	0.48	0.90
		loglinear	0.01	0.84	0.70	0.84
ser	19 obs.	linear	0.01	0.72	0.19	0.72
		loglinear	0.01	0.76	0.01	0.76
	6 obs.	linear	0.36	0.20	0.58	0.20
		loglinear	0.31	0.25	0.01	0.25
year	19 obs.	linear	0.56	0.01	0.01	0.01
		loglinear	0.49	0.01	0.01	0.01
	6 obs.	linear	0.01	0.87	0.88	0.87
		loglinear	0.03	0.76	0.76	0.76
t/w	19 obs.	linear	0.17	0.08	0.80	0.08
		loglinear	0.30	0.01	0.01	0.01
	6 obs.	linear	0.03	0.73	0.21	0.73
		loglinear	0.02	0.78	0.01	0.78

3) Models with 1 variable

VAR'	DATA	FUNCTION	R^2	PROB > F	PROB > $\lvert t_{b1} \rvert$	PROB > $\lvert t_{b2} \rvert$
span	19 obs.	linear	0.27	0.02	0.39	0.02
		loglinear	0.10	0.20	0.47	0.20
	6 obs.	linear	0.42	0.17	0.95	0.17
		loglinear	0.37	0.20	0.58	0.20
thrust	19 obs.	linear	0.65	0.01	0.43	0.01
		loglinear	0.70	0.01	0.01	0.01
	6 obs.	linear	0.15	0.45	0.80	0.45
		loglinear	0.16	0.44	0.85	0.44
weight	19 obs.	linear	0.62	0.01	0.19	0.01
		loglinear	0.64	0.01	0.01	0.01
	6 obs.	linear	0.23	0.34	0.64	0.34
		loglinear	0.16	0.43	0.96	0.43
speed	19 obs.	linear	0.41	0.01	0.29	0.01
		loglinear	0.55	0.01	0.01	0.01
	6 obs.	linear	0.01	0.89	0.48	0.90
		loglinear	0.01	0.84	0.70	0.84
ser	19 obs.	linear	0.01	0.72	0.19	0.72
		loglinear	0.01	0.76	0.01	0.76
	6 obs.	linear	0.36	0.20	0.58	0.20
		loglinear	0.31	0.25	0.01	0.25
year	19 obs.	linear	0.56	0.01	0.01	0.01
		loglinear	0.49	0.01	0.01	0.01
	6 obs.	linear	0.01	0.87	0.88	0.87
		loglinear	0.03	0.76	0.76	0.76
t/w	19 obs.	linear	0.17	0.08	0.80	0.08
		loglinear	0.30	0.01	0.01	0.01
	6 obs.	linear	0.03	0.73	0.21	0.73
		loglinear	0.02	0.78	0.01	0.78

APPENDIX D
CHOW TEST

H_0 : The models based on 6 and 13 observations came from the same population.

	model 1	model 2	model 3
number of observations (n)	6	13	19
number of parameters (k)	3	3	3
residual sum of squares (RSS)	17.7728	78.0055	269.49

$$F = \frac{(RSS_3 - (RSS_1 + RSS_2))(n - 2k)}{(RSS_1 + RSS_2)(k)} \sim F(k,n_3\text{-}2k)$$

Thus,

$$F = \frac{(269.49 - (17.7728 + 78.0055))(19 - 6)}{(17.7728 + 78.0055)(3)}$$

$$= 7.859$$

Then, the critical value is

$$F_{0.95}(3,13) = 3.415 < 7.859$$

Therefore, the null hypothesis that the model based on 6 and 13 observations came from the same population is rejected.

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